ference in the maximal values of the amplitude is not more than 3% (t = 120). It is seen from an analysis of the perturbation oscillogram obtained numerically that the amplitude behind the front oscillates relative to the mean value $w = q_{*}x/4h$ determined from (6).

Graphs of the deflection computed for a local oscillating load are represented in Figs. 5 and 6. The numerical results are in good agreement with the analytic results: for $\omega_0 < 1$ the perturbation amplitude does not grow with time; for $\omega_0 = 1$ (Fig. 5a is the diagram for t = 90 and b is the oscillogram at the point x = 0) the perturbation envelope in the neighborhood of x = 0 increases in proportion to $t^{3/4}$; for $\omega_0 > 1$ (Fig. 6, $\omega_0 = \sqrt{2}$) the perturbations propagate with an amplitude oscillating relative to the mean value (dashed line) that is found from (7).

Comparing the numerical and analytic solutions describing bending resonance wave propagation in a cylindrical shell and a rod on an elastic basis shows that the asymptotics obtained determine, with good accuracy, the fundamental perturbations in a system formed after a finite time interval.

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CALCULATION OF STRAINS FOR BRITTLE MATERIALS TAKING INTO ACCOUNT LIMITING FAILURE

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In order to describe the strain properties of heterogeneous materials there is currently extensive use of a model for an elastic material weakened by a large number of cracks [1-9].

The aim of the present work is to construct a system of fundamental equations for computing the strain properties of brittle materials based on development of a model for a cracked material suggested in [3, 4, 9] taking account of crack growth during deformation.

1. We consider development of an isolated shear crack. Shear crack propagation in a plane arrangement was studied in [6-8] where it was noted that during loading in the end zones of a shear crack separation cracks occur growing in the general case along a curvilinear trajectory.

Experiments [8] show that curvature of a growing separation crack occurs directly adjacent to the end zone of a shear crack. Subsequently, independent of the direction for the plane of a shear crack growth of a separation crack occurs in a plane perpendicular to the direction of least compressive stress.

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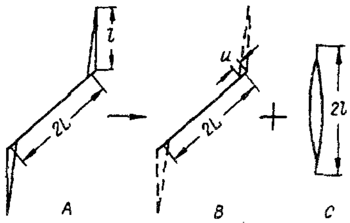

Fig. 1

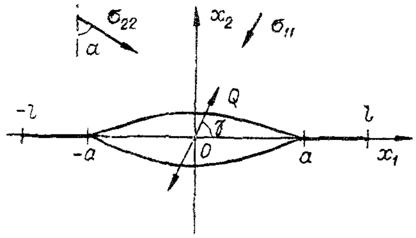

Fig. 2

In contrast to [9], in the present work an individual crack is modeled by a three-component discontinuity consisting of a shear crack 2L long and two centrally symmetrical rectilinear separation cracks with initial length ℓ (Fig. 1).

The macroscopic strain tensor for a cracked material in the method of indirect averaging of the field over the volume [2] is determined through the displacement in crack edges averaged for surfaces of a static assembly of discontinuities. Since local displacements along the discontinuity do not enter directly into the final result, then in order to simplify calculations with retention of all characteristic features of the solution for the original problem average displacements of the surface for a three-component crack are assumed to be represented in the form of superposition of the average displacements of surfaces for two cracks (Fig. 1). Continuity of displacements for the discontinuity edges at points of inflection provides conformity of boundary conditions in sections B and C.

First we consider crack B in an unbounded elastic material with Young's modulus E_0 and Poisson's ratio ν_0. At the center of this crack we place a coordinate system xOy so that axis Ox is directed along the discontinuity. The stressed state at infinity is assumed to be uniform, and principal stresses are σ_{11} and σ_{22}. For this stressed state crack orientation is governed by angle θ between the principal axis corresponding to σ_{22} and axis Oy. We stay with the case of compressive stresses $\sigma_{22} < \sigma_{11} \leq 0$.

We assume that the crack edges react according to the Coulomb rule [3]

$$\tau_{-} = \tau_0 + \mu\sigma_n, \tag{1.1}$$

where μ is friction coefficient, τ_0 is cohesion; σ_n is normal stress. The taking account of the continuity condition for displacements at points of inflection for the discontinuity displacements of the closed crack surface under the action of compressive stresses are found from the expressions

$$v_1(x,\ 0) = w_1(x,\ 0) + u,\ v_2(x,\ 0) = 0. \tag{1.2}$$

Here u is determined by edge displacement for crack C (Fig. 1), and w_1 is determined from solution of the problem of a rectilinear section with reacting edges [10]:

$$w_1(x, 0) = \pi D\sqrt{L^2 - x^2}\,\tau_{+},\quad D = \frac{(1+\varkappa)(1+\nu_0)}{2\pi E_0},$$

where 2L is length of crack B; τ_+ is effective shear stress characterizing the mutual displacement of closed crack edges and it equals $\tau_+ = |\tau| - \tau_-$ with $|\tau| > \tau_-$; with $|\tau| \leq \tau_-$, the crack edge is in a cohesive condition and $\tau_+ = 0$; $\varkappa = (3 - \nu_0)/(1 + \nu_0)$ for a general plane stressed state; $\varkappa = 3 - 4\nu_0$ for plane strain.

Now we consider a crack modeling separation cracks at the tips of a shear crack. We select a coordinate system so that the original coincides with the center of the crack and the section lies along axis Ox_1 (Fig. 2). For this stressed state crack orientation is prescribed by angle α between the principal axis corresponding to σ_{22} and axis Ox_2.

In order to calculate the effect of section B on separation crack C in a limiting condition it is necessary to consider stresses arising at the surface of crack C with advance of the edges of section B. The nature of tensile stresses arising at the surface of crack C may only be established from the analysis of singular integral equations for the problem of a three-component discontinuity [6]. In order to simplify calculations in this work, the effect of a shear crack is considered approximately as in [8, 11] by introducing a concentrated force into the boundary condition in section C. It is assumed that concentrated tensile forces are proportional to effective shear stress τ_+ and they are directed along the surface of section C (Fig. 2, where γ is the angle between shear and separation sections in a three-component crack).

Model representation of tensile stresses in the form of concentrated forces leads to unbounded displacement in the vicinity of the applied force which is logarithmic in character [12]. However, as demonstrated with numerical modeling of a three-component discontinuity [6], crack edge displacements in the area of a break are finite. On the other hand, in the approach suggested in [2] the macroscopic strain tensor for a cracked material is expressed in terms of displacements averaged over the crack surface and the contribution of concentrated forces to material strain will be finite.

Under the action of compressive stresses σ_{11} and σ_{22} the edges of a separation crack may join together smoothly and react in areas $[-\ell, -a]$ and $[a, \ell]$. Boundaries of the region $[-a, a]$ are governed by the equation $K_1(\pm a) = 0$ [10, 12] (K_1 is separation stress intensity factor).

We consider the class of loading trajectory which, at the separation crack surface zones of edge of joining do not arise; it is of practical interest since, in fact, under these conditions crack growth occurs with loading. Then the boundary problem for a separation crack has the form

$$\begin{gathered} u_1^+(x_1, 0) = u_1^-(x_1, 0) \quad (|x_1| \geqslant l), \quad u_2^+(x_1, 0) = u_2^-(x_1, 0) \quad (|x_1| \geqslant l), \\ \sigma_{22}^{\pm}(x_1, 0) = Q \sin^2\gamma\,\delta(x_1) - \sigma_1 \quad (|x_1| < l), \quad \sigma_{12}^{\pm}(x_1, 0) = \\ = Q \sin\gamma\cos\gamma\,\delta(x_1) + \tau_1 \quad (|x_1| < l), \\ \sigma_1 = |\sigma_{11}|\cos^2\alpha + |\sigma_{22}|\sin^2\alpha, \; Q = 2L\tau_+, \; \tau_1 = (\sigma_{11} - \sigma_{22})\sin\alpha\cos\alpha, \end{gathered} \tag{1.3}$$

where $u_i^{\pm}$ are displacements at the upper and lower edges of the discontinuity; $\delta(x)$ is the delta function.

The distribution of displacements along axes Ox_1 and Ox_2 is determined according to [12] by the relationship

$$\begin{gathered} u_i = u_i^+ - u_i^- = D\int_{-l}^{l} \sigma_{i2}^{\pm}(t, 0)\sqrt{l^2 - t^2}\,\times \\ \times \int_{-l}^{x_1} \frac{dt\,d\eta}{(t-\eta)\sqrt{l^2-\eta^2}} \quad (i = 1, 2, |x_1| < l), \end{gathered} \tag{1.4}$$

by the use of which for boundary problem (1.3) we obtain the distribution of displacements along axes Ox_1 and Ox_2:

$$\begin{gathered} u_2 = D\left(\frac{2}{\pi}L\tau_+\sin^2\gamma\ln\left|\frac{\sqrt{l-x_1}+\sqrt{l+x_1}}{\sqrt{l-x_1}-\sqrt{l+x_1}}\right| - \sigma_1\sqrt{l^2-x_1^2}\right), \\ u_1 = D\left(\frac{L}{\pi}\tau_+\sin 2\gamma\ln\left|\frac{\sqrt{l-x_1}+\sqrt{l+x_1}}{\sqrt{l-x_1}-\sqrt{l+x_1}}\right| + \tau_1\sqrt{l^2-x_1^2}\right). \end{gathered} \tag{1.5}$$

Expressions for stress intensity factor for (1.3) characterizing the singular solution at points $x_1 = \pm\ell$ according to [12] have the form

$$K_1 = \sqrt{\pi l}\left(\frac{2\tau_+ L}{\pi l}\sin^2\gamma - \sigma_1\right), \quad K_2 = \sqrt{\pi l}\left(\frac{2\tau_+ L}{\pi l}\sin\gamma\cos\gamma + \tau_1\right) \tag{1.6}$$

(K_2 is intensity factor for transverse shear).

The limiting equilibrium condition of the crack tip in the general case is determined by the inequality [13]

$$(K_1/K_{1c})^2 + (K_2/K_{2c})^2 < 1 \tag{1.7}$$

(K_{1c} and K_{2c} are constants connected with material failure).

By summarizing results [14] for a complex type of loading and taking account of inequality (1.7), in order to describe the change with time of crack size we use the relationship

$$v=\begin{cases}0, & K<K_{1c},\\ v_0\Phi(K), & K_{1c}\leqslant K<K^*,\\ v_0, & K\geqslant K^*,\end{cases} \tag{1.8}$$

where $K=\sqrt{K_1^2+(K_{1c}K_2/K_{2c})^2}$; v_0 is limiting crack growth rate; K^* is failure threshold with the limiting velocity; $\Phi(K)=A_1\exp(\lambda K)$ for dry rocks and $\Phi(K)=A_2K^n$ for impregnated materials [15]; A_1, A_2, n, and λ are material constants.

By means of expression (1.6) conditions are found for formation of the contact region at the surface of a separation crack and its boundaries:

$$a=\begin{cases}\dfrac{2L}{\pi\sigma_1}\tau_+\sin^2\gamma, & \tau_+<\dfrac{\pi l\sigma_1}{2L\sin^2\gamma},\\ l, & \tau_+\geqslant\dfrac{\pi l\sigma_1}{2L\sin^2\gamma}.\end{cases} \tag{1.9}$$

Relationships (1.5), (1.6), and (1.8) make it possible to work out separation crack extension taking account of the possibilities of a change in its length with loading with absence of a contact zone at the discontinuity surface determined by expression (1.9). As can be seen from (1.6), stress intensity factors decrease with an increase in separation crack length at constant load and in accordance with (1.8) cracks may move into a stable condition which agrees with the results in [6, 7].

2. We consider deformation of an elastic material weakened by identical isolated three-component discontinuities whose problem of edge displacement was solved in Sec. 1. By means of the method in [2], the increase in macroscopic strain tensor in the suggested model is presented in the form

$$\begin{gathered}\delta\varepsilon_{ik}=\delta\varepsilon_{ik}^0+\frac{1}{2}\int(n_i\delta U_k+n_k\delta U_i)F(Y)\,dY+\frac{1}{2}\int(m_i\delta W_k+m_k\delta W_i)\times\\ \times F(Y)\,dY+\frac{1}{2}\int(n_iU_k+n_kU_i+m_iW_k+m_kW_i)\,\delta F(Y)\,dY.\end{gathered} \tag{2.1}$$

Here $\varepsilon_{ik}{}^0$ is strain tensor for a solid linearly elastic material; n_i and m_i are components of normal unit vectors to the surfaces of discontinuities B and C; U_i and W_i are vector components for the average jump in edge displacements for a closed and open crack; F(Y) is the crack distribution function for a collection of parameters for Y introduced in [2]. Crack sizes and their spatial orientation prescribed for a plane stressed state by angle α and γ are taken as parameters of Y.

It is assumed that crack density does not change during loading and the distribution function is known up to load $f(L, \ell, \alpha, \gamma)$. Then, considering that in the model in question separation cracks remain rectilinear during loading and the distribution function is written as

$$F(Y)=f\left(L,\ l+\int_0^t v(l,\alpha,\gamma)\,dt',\ \gamma,\ \alpha\right) \tag{2.2}$$

[v is separation crack growth rate determined by relationship (1.8)]. Using (1.5) by carrying out averaging for the separation crack surface, the components of vector W_i are presented in the form

$$W_2=Dl^2\left(4\tau_+\frac{L}{l}\sin^2\gamma-\pi\sigma_1\right),\quad W_1=Dl^2\left(2\tau_+\frac{L}{l}\sin 2\gamma+\pi\tau_1\right). \tag{2.3}$$

With averaging of vector components for the jump in displacements v_i, according to expression (1.2) it is necessary to carry out averaging of displacements at points of inflection for the three-component discontinuity. The average displacement at points of inflection is assumed to be the limit

$$U(0)=\lim_{z\to 0}\left(\frac{1}{2z}\int_{-z}^{z}(u_2(x_1,0)\cos\gamma+u_1(x_1,0)\sin\gamma)\,dx_1\right). \tag{2.4}$$

On the basis of relationships (1.2), (1.5), and (2.4) we find the vector component for the average jump in displacements of the edges of crack B:

$$\begin{gathered}U_1=DL^2(\pi\tau_+ +2X)\,n_2,\quad U_2=DL^2(\pi\tau_+ +2X)\,n_1,\\ X=\tau_+\sin\gamma\sin 2\gamma-\frac{l}{L}(\sigma_1\cos\gamma-\tau_1\sin\gamma).\end{gathered} \tag{2.5}$$

As can be seen from (2.3) and (2.5) components of vectors for average jumps in displacements are limiting values. Substitution of relationships (2.2), (2.3), and (2.5) in Eq. (2.1) determines the strain tensor for a cracked material. Integration in (2.1) should be carried out within limits where mutual displacement of crack edges occurs, $\gamma_1 \le \gamma \le \gamma_2$, characterized by relationship (1.1). In view of symmetry for the problem, sectors $0 \le \gamma \le \pi/2$, $0 \le \alpha \le \pi/2$ are considered.

Thus, the strain equation for a cracked material taking account of the development of cracking has the form

$$\dot{\varepsilon}_{ik}=\dot{\varepsilon}^0_{ik}+A_{ikjl}\dot{\sigma}_{jl}+B_{ikjl}\sigma_{jl} \tag{2.6}$$

[tensors $A_{ijk\ell}$ and $B_{ikj\ell}$ are found from relationships (1.1), (1.6), (1.8), (2.1)-(2.3), and (2.5)].

Tensor components $B_{ikj\ell}$ obtained for the form of stressed state and initial crack distribution $f = (1/\pi)\delta(L - L_0)\delta(\ell)$ have the form

$$\begin{gathered}
B_{1111}=2\pi\sin\alpha\int_{\xi_1}^{\xi_2}C\frac{v}{l}\,d\gamma+\int_{\xi_1}^{\xi_2}KCv\cos\gamma\sin(\alpha-\gamma)\times\\
\times\left(\sqrt{1+\mu^2}\sin(2\gamma-\beta)+\mu\right)d\gamma,\\
B_{2222}=2\pi\cos\alpha\int_{\xi_1}^{\xi_2}C\frac{v}{l}\,d\gamma+\int_{\xi_1}^{\xi_2}KCv\sin\gamma\sin(\alpha-\gamma)\left(\sqrt{1+\mu^2}\sin(2\gamma-\beta)-\mu\right)d\gamma,\\
B_{1122}=-\sin\alpha\int_{\xi_1}^{\xi_2}KCv\cos\gamma\sin(\alpha-\gamma)\left(\sqrt{1+\mu^2}\sin(2\gamma-\beta)-\mu\right)d\gamma,\\
B_{2211}=-\cos\alpha\int_{\xi_1}^{\xi_2}KCv\sin\gamma\sin(\alpha-\gamma)\left(\sqrt{1+\mu^2}\sin(2\gamma-\beta)+\mu\right)d\gamma,\\
B_{1211}=\frac{\pi}{2}\sin 2\alpha\int_{\xi_1}^{\xi_2}C\frac{v}{l}\,d\gamma+\frac{1}{2}\int_{\xi_1}^{\xi_2}KCv\sin(\alpha-\gamma)\cos(\alpha+\gamma)\times\\
\times\left(\sqrt{1+\mu^2}\sin(2\gamma-\beta)+\mu\right)d\gamma,\\
B_{1222}=\frac{\pi}{2}\sin 2\alpha\int_{\xi_1}^{\xi_2}C\frac{v}{l}\,d\gamma-\frac{1}{2}\int_{\xi_1}^{\xi_2}KCv\cos(\alpha+\gamma)\sin(\alpha-\gamma)\times\\
\times\left(\sqrt{1+\mu^2}\sin(2\gamma-\beta)-\mu\right)d\gamma,\\
K=\frac{5L_0}{l},\quad C=\frac{\pi}{2}DNl^2,\quad \beta=\operatorname{arctg}\mu,
\end{gathered} \tag{2.7}$$

where (ξ_1, ξ_2) is the sector of crack growth determined by relationship (1.8); N is the crack concentration.

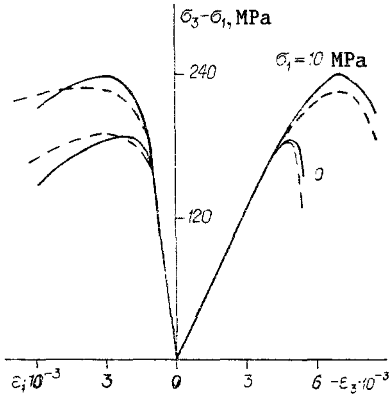

Fig. 3

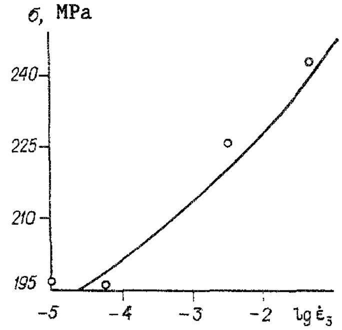

Fig. 4

Tensor components $A_{ikj\ell}$ may be obtained from (1.1), (1.6), (1.8), (2.1)-(2.3), and (2.5), but in view of the cumbersome nature of expressions they are not provided. Relationships (2.3), (2.5), and (2.7) were found in an approximation of noninteracting cracks ($NL^2 \ll 1$, $N\ell^2 \ll 1$). With consideration of cracking during loading, the question arises of the necessity for considering the reaction between cracks. With presence in the material of a set of main cracks their nonsteady-state reaction may markedly affect failure kinetics [16, 17] leading to stopping of some of the cracks and formation of crack-leaders.

In considering materials weakened by cracks with similar parameters for which in the initial stages quasivolumetric failure is typical without the occurrence of clearly defined crack leaders, it is suggested that reaction between cracks up to the instant of multiple intersection of cracks ($N\ell^2 \sim 1$) is considered approximately by means of effective elasticity moduli E and ν. It is assumed that an isolated crack is in the material with elasticity moduli E and ν governed by other cracks. By effective elasticity moduli for the stressed state in question we understand derivatives $E = d\sigma_{22}/d\varepsilon_{22}$ and $\nu = -d\varepsilon_{11}/d\varepsilon_{22}$. Within the limits of this approach the effect of material anisotropy on extension of isolated crack edges is assumed to be averaged.

3. In order to compare the results obtained with experimental data, a calculation was carried out for strain curves for actual brittle materials with a complex stressed state used in order to study limiting material characteristics.

In the theoretical model suggested with calculation of tensor components $A_{ijk\ell}$ and $B_{ikj\ell}$ in Eq. (2.6) for actual brittle materials, three types of parameter are used: characterizing growth of an isolated crack K_{1c}, K_{2c}, K^*, α, v_0 and reaction of the edges μ, τ_0; elasticity moduli for a solid material E_0 and ν_0; initial cracking characteristics $NL_0{}^2$ and L_0.

Parameters of the first type are found from the results of independent experiments for studying the advance of edges for an isolated crack and its propagation in brittle materials. According to data in [15, 18] the following values for granite are found: $\tau_0 = 0$, $\mu = 0.68$, $K_{1c} = 1.07$ MPa·m$^{1/2}$, $K_{2c} = 1.36$ MPa·m$^{1/2}$, $K^* = 2.15$ MPa·m$^{1/2}$, $\alpha = 6.7$ (MPa·m$^{1/2}$)$^{-1}$, and v_0 = 740 m/sec.

Parameters of the second type were determined by means of strain curves obtained for separate fixed loading regimes in the stage up to the start of material failure. With existence of sufficient side pressure in a complex stressed state there is crack closing and from the slope of the linear sections of the strain curves E_0 and ν_0 are found. For granite, E_0 = 65,000 MPa and $\nu_0 = 0.18$ were used in experiments [9].

Parameters of the third type were determined from experimental data for material axial compression. For $\ell = 0$, advance of the edges of closed cracks with uniaxial compression in accordance with (1.1) proceeds in the range of angles $\arctan\mu \le \gamma \le \pi/2$ and effective elasticity moduli remain constant up to the instant of the start of the development of cracking. The difference $E - E_0$ (E is Young's modulus for uniaxial compression) determines the initial stage of material cracking $\Omega_0 = NL_0{}^2$.

Deviation from a linear relationship in the case of uniaxial compression is connected with the start of crack growth. Therefore, the average initial crack size is found from the value of limit of proportionality for stresses by means of crack growth criterion (1.8). For granite, $\Omega_0 = 0.2$, $L_0 = 0.9$ mm.

For a complex stressed state in calculating tensors $A_{ikj\ell}$ and $B_{ikj\ell}$ integration in accordance with (2.1) was carried out for angles characterizing crack spatial orientation.

Strain properties for granite in a complex stressed state were calculated by means of the parameters provided and Eq. (2.6). Presented in Fig. 3 are strain curves obtained with different values of side pressure (broken lines are the experiment [19]). In the approach developed a study was made of the dependence of limiting strength of granite in a complex stressed state on loading rate (Fig. 4). The good conformity of the results obtained with experimental data makes it possible to conclude that, in spite of simplifications, Eq. (2.6) may be used in order to describe the strain properties of brittle materials in a complex stressed state in the stage of crack growth.

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